NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3530

MINIMUM WAVE DRAG FOR ARBITRARY ARRANGEMENTS OF WINGS AND BODIES

By Robert T. Jones

Ames Aeronautical Laboratory
Moffett Field, Calif.

Washington
February 1956

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3530

MINIMUM WAVE DRAG FOR ARBITRARY ARRANGEMENTS OF WINGS AND BODIES

By Robert T. Jones

SUMMARY

Studies of various arrangements of wings and bodies designed to provide favorable wave interference at supersonic speeds lead to the problem of determining the minimum possible value of the wave resistance obtainable by any disposition of the elements of an aircraft within a definitely prescribed region. Under the assumptions that the total lift and the total volume of the aircraft are given, conditions that must be satisfied if the drag is to be a minimum are found. For arbitrary regions the minimum value can be estimated by a simple formula giving a lower bound.

DISCUSSION

In 1935 Busemann (ref. 1) showed that the wave drag of two airfoils could be canceled by reflection. Later Ferrari (ref. 2) showed that the drag of a body of revolution could be canceled by the addition of a ring airfoil to catch the wave from the nose and reflect it back to the tail. Even if the investigation is limited to such completely self-contained wave systems, these examples are only two of an infinite number of possibilities.

The examples in which the wave cancellation is complete are, however, limited to systems in which the net lift and lateral force are zero. Nevertheless, examples cited by Ferri (ref. 3) and by Graham (ref. 4) show that the wave drag associated with the lift can be diminished by various three-dimensional arrangements of wings and bodies. These examples lead to a search for some general statements or criteria regarding the wave drag of such three-dimensional arrangements.

In order to put the question in a definite form it will be assumed that the airfoils and bodies are contained in the interior of a definite three-dimensional region R. The total lift L and the volume v are assumed to be given. It is supposed that the wave drag D depends somehow on the distribution of the lift and the volume throughout R and that with distributions of a certain family (called "optimum" ones) the drag will have a minimum value. It is desired to find the optimum distributions, or the conditions determining them, and the value of the minimum drag. Problems of this type have been considered by E. W. Graham and his colleagues who give, for example, the optimum distribution of lift within a spherical region.

If the region R is restricted to the plan form S of a planar wing, then problems of a type previously discussed by the present writer are obtained (refs. 5 and 6). In connection with the latter problems it was found that all distributions of lift or volume satisfying the given requirements could be characterized by relatively simple conditions. The present paper describes briefly the extension of these conditions to three-dimensional regions and the additional conditions required.

As is usual in linearized-flow problems it will be assumed that the disturbance field of the airfoils and bodies can be produced by the action of a distribution of sources and "lifting elements" or horseshoe vortices. One of the difficulties associated with these problems is the determination of the actual geometric shapes produced by the distribution of singularities. In the present analysis the relation between the body shapes and the singularities is not known nor determined in detail. For slender bodies or thin airfoils closed within the region R it can be assumed that the total volume is proportional to the first moment of the source distribution with respect to a plane perpendicular to the flight direction, whereas the total lift is proportional to the total strength of the lifting elements.

Suppose a region R together with a distribution of singularities such as sources or lifting vortices is given. (See fig. 1.) Then by Hayes' theorem (ref. 7), the drag will be unchanged by a reversal of the whole system. (The geometry of the flow, including that of the airfoils and bodies, will be changed by the reversal but the total lift and the total volume will not.) Then the drag may be computed by means of a fictitious "combined disturbance field" obtained by superimposing the disturbances in the forward and the reversed motion. The perturbation velocities in this combined field may be denoted by

$$2\bar{u} = u_f + u_r$$

$$2\bar{v} = v_f + v_r$$

$$2\bar{w} = w_f + w_r$$

An arrangement of sources or lifting elements or their combination which yields the minimum drag is then characterized by the conditions

$$\left.\begin{aligned}\bar{w} &= \text{Constant}\\ \bar{v} &= 0\\ \frac{\partial\bar{u}}{\partial x} &= \text{Constant}\end{aligned}\right\} \tag{1}$$

throughout R.

If conditions (1) are satisfied, then the integrated drag of the whole system will be given simply by

$$D_{MIN} = L\frac{\bar{w}}{V} + \nu\rho V\frac{\partial\bar{u}}{\partial x}$$

The first term on the right-hand side of this expression will be recognized as the drag arising from a rearward inclination of the lift vector, whereas the second term is simply the product of the volume and the constant gradient of pressure in the combined flow field.

These conditions may be verified by making use of a "mutual drag relation," essentially similar to the well-known Ursell-Ward reciprocal relation, which connects the drag of any two interfering distributions of singularities in the combined flow field. According to this relation the drag of distribution A caused by the interference of a second distribution B is equal to the drag added to B by the interference of A. Now let A be a distribution within R_A satisfying conditions (1). For B select a distribution having zero total lift and zero total volume. If R_B is contained within R_A, then the addition of B will amount simply to a redistribution, without changing the total lift L or the volume ν, of A. The drag of A + B may then be written in shorthand notation

$$D(A + B) = D_{AA} + D_{AB} + D_{BA} + D_{BB}$$

Then, since by the mutual drag relation D_{AB} is equal to D_{BA}, this equation may be written as

$$D(A + B) = D_{AA} + 2D_{BA} + D_{BB}$$

Here D_{BA} is the drag of B in the combined disturbance field of A. Since $\bar{w}$ = Constant, $\bar{v} = 0$, and $\frac{\partial\bar{u}}{\partial x}$ = Constant in R_A, this interference drag may be written as

$$D_{BA} = L_B \frac{\bar{w}_A}{V} + \nu_B \rho V \left(\frac{\partial \bar{u}}{\partial x}\right)_A$$

However, since L_B and ν_B are zero, D_{BA} vanishes and the added drag is that of distribution B alone or D_{BB}. Now the drag of a system alone, that is, without interference, cannot be negative; hence, $D(A + B)$ cannot be less than $D(A)$ under conditions (1).

On the other hand, suppose, for example, that the sidewash $\bar{v}_A$ were not zero. A distribution of lateral forces could then be found which would result in a negative interference drag, dominating the quadratic term D_{BB}, so that the total drag could be reduced. Hence, if the drag of distribution A actually is a minimum value, conditions (1) must be complied with.

Since $w = \frac{\partial \phi}{\partial z}$, $v = \frac{\partial \phi}{\partial y}$, and $\frac{\partial u}{\partial x} = \frac{\partial^2 \phi}{\partial x^2}$, it can be seen that conditions (1) do not agree with the linearized flow equation

$$\left(1 - M^2\right)\phi_{xx} + \phi_{yy} + \phi_{zz} = 0$$

in general, but only if $\frac{\partial \bar{u}}{\partial x} = 0$. Since $\frac{\partial \bar{u}}{\partial x}$ is proportional to the drag per unit volume, one concludes that the drag cannot be minimized in an absolute sense unless the drag associated with the volume of the system is zero (or unless the distribution of singularities is continuous throughout R). Examples such as the Busemann biplane satisfy the former condition, namely, $\frac{\partial \bar{u}}{\partial x} = 0$.

As Graham et al. have pointed out, distributions of the sort being considered here are not unique, since other solutions such as those shown in figure 2 may be added to them without changing the lift or the drag.

It is interesting to note that conditions analogous to the conditions $\bar{w} =$ Constant and $\bar{v} = 0$ were found by Munk in connection with the vortex drag of lifting systems at subsonic speeds. In that problem the conditions apply to the two-dimensional motion associated with the trace of the wing system in the Trefftz plane. If the idea of superimposed flow fields is utilized in the subsonic problem, one finds that the cylindrical flow associated with the Trefftz plane extends along the whole flight path and hence includes the region R. Conditions (1) thus apply at both subsonic and supersonic speeds, but are unnecessarily restrictive at subsonic speeds.

Munk's conditions of constant downwash and zero sidewash were used by Hemke (ref. 8) to determine the effectiveness of end plates in reducing the vortex drag of a wing at low speeds. It will be interesting to see how the condition $\bar{v} = 0$ might be used to determine an optimum setting and camber for such a surface under more general conditions. This application is illustrated in figure 3 for an end plate on the tip of a wing.

With the wing in forward motion, the lateral velocity v_f at the surface of the end plate is simply the lateral slope of the fin surface times the stream velocity. The condition $\bar{v} = 0$ implies that $v_r = -v_f$, and this condition is obviously satisfied by keeping the geometry of the fin fixed when the flow is reversed. At the same time, however, recall that the distribution of lift and lateral force must be kept the same in forward and reversed flow. Hence, in order to achieve the minimum drag one must find the particular camber and setting of the fin that will yield the same distribution of lateral force for either direction of motion. At first it seems impossible to satisfy such a requirement since, for example, the direction of lift of an inclined surface is usually reversed by reversing the direction of flow. However, the form of the adjacent wing surface must, in general, change with reversal, since $\bar{w} \neq 0$ and since the lift distribution on the wing must remain unchanged. Then it is evident that the conditions might be satisfied if the pressures on the fin surface were dominated by the wing pressures through interference.

It must be admitted that the considerations have thus far been rather abstract. A more concrete result would yield the actual magnitudes of the minimum drag associated with various regions. Such results for distributions of lift in spherical and ellipsoidal regions have been given in reference 4. A somewhat more general result, applicable to arbitrary regions R, can be obtained if merely a lower bound for the wave drag is sought rather than the actual minimum value. Since this lower bound coincides with the minimum value in the examples found thus far, it may be taken as an approximation to the actual drag in many cases.

To obtain such a lower bound, we may use Hayes' formula (ref. 7) — or the formula of Lomax (ref. 9), which expresses the drag more directly in terms of areas and pressures intercepted by characteristic planes. By utilizing Hayes' method of equivalent positions (ref. 7) or the present writer's method of superimposing plane waves (ref. 6), one can construct, at each angle θ, three equivalent linear distributions, namely, a volume distribution, a lift distribution, and a side-force distribution. By a harmonic analysis (ref. 10) it is possible to show that the drag associated with the leading term in the expansion of the lift distribution $l(x)$, proportional to the total lift L, cannot be diminished by interference. The possibility, already known, that the drag associated with the volume can be eliminated appears in this analysis. Hence, for the lower bound the value given by the first term in the expansion of the lift distribution is used. This step amounts to the assumption that each "lifting line" obtained by integrating the spatial lift distribution over the intersecting Mach planes is elliptically loaded. For a single elliptically loaded lifting line parallel to the flight direction, the wave drag is

$$D_{WAVE} = \frac{M^2 - 1}{2} \frac{L^2}{\pi q l^2} \tag{2}$$

where l is the length of the line. For the whole region R the following is obtained

$$D_{WAVE} \geqq \frac{M^2 - 1}{2} \frac{L^2}{\pi q \bar{l}^2} \tag{3}$$

where

$$\frac{1}{\bar{l}^2} = \frac{1}{\pi} \int_0^{2\pi} \frac{\sin^2\theta \, d\theta}{\left[l(\theta)\right]^2} \tag{4}$$

and $l(\theta)$ is the projected length of R as defined in figure 4, with $\beta = \sqrt{M^2 - 1}$.

It will be evident from equation (3) that the wave drag depends inversely on the square of an average projected length of the airfoil system - just as the vortex drag depends inversely on the square of the span. However, because of the weighting factor $\sin^2\theta$ the lateral dimensions of R are relatively unimportant compared to the dimension, or length, along the flight direction. Figure 5 shows the magnitude of the error made by using the actual length l and equation (2) for the wave drag of several lifting surfaces.

Generally speaking, the losses associated with the production of a given force in a frictionless fluid are diminished by increasing the area involved in the production of the force and diminishing the pressure. Thus the wave drag is diminished by making the "area" $\bar{l}^2$ as large as possible. The vortex drag is diminished by making the square of the span as large as possible. On the other hand, to diminish the friction drag the actual area S of the wing system must be made as small as possible.

At subsonic speeds the conditions are satisfied by making b^2 large compared with S or using a wing of high aspect ratio. It is a matter of ordinary observation to see that this condition determines the rather special form of subsonic aircraft. At supersonic speeds, a large value of the "longitudinal aspect ratio" $\frac{\bar{l}^2}{S}$ is needed in addition.

At subsonic speeds, the elliptically loaded lifting line achieves the minimum value of the pressure drag for the whole area covered by the wake of the lifting line. At supersonic speeds such a lifting line develops, according to linear theory, an infinite drag. However, if the line is yawed behind the Mach angle the drag is finite and is actually the smallest value obtainable by any distribution within the region of the parallelogram ABCD shown in figure 6. Such an oblique lifting line maximizes both $\frac{b^2}{S}$ and $\frac{\bar{l}^2}{S}$ simultaneously. At moderate supersonic Mach numbers, the results obtained with a V-shaped lifting line - approximating a swept wing - are nearly as good.

When a wing is made narrower so as to approach a "lifting line" while maintaining a fixed total lift, the lifting pressure must increase. Eventually the pressure, or the lift coefficient, will exceed the limitation imposed by the small-disturbance theory, or flow separation will occur. Beyond this point increases of aspect ratio either laterally or longitudinally will not necessarily increase the lift-drag ratio.

Ames Aeronautical Laboratory
National Advisory Committee for Aeronautics
Moffett Field, Calif., Nov. 1, 1955

REFERENCES

1. Busemann, A.: Aerodynamischer Auftrieb bei Überschallgeschwindigkeit (Supplement). Fifth Volta Congress, Reale Academia D'Italia, 1935.

2. Ferrari, C.: Campi di corrente impersonora attorno a solidi di rivoluzione. L'Aerotecnica, vol. XVII, fasc. 6, June 1937, pp. 507-518. (Also available as Brown Univ. Graduate Div. of Applied Math. Trans. 3965a.)

3. Ferri, Antonio: Recent Work in Supersonic and Hypersonic Aerodynamics at the Polytechnic Institute of Brooklyn. Paper given at the conference on High Speed Aeronautics held by the Polytechnic Institute of Brooklyn, Jan. 22, 1955.

4. Graham, E. W., Lagerstrom, P. A., Licher, R. M., and Beane, B. J.: Theoretical Investigation of the Drag of Generalized Aircraft Configurations in Supersonic Flow. Rep. No. SM-19181, Douglas Aircraft, Co., Inc., July 1955.

5. Jones, Robert T.: The Minimum Drag of Thin Wings in Frictionless Flow. Jour. Aero. Sci., vol. 18, no. 2, Feb. 1951, pp. 75-81.

6. Jones, Robert T.: Theoretical Determination of the Minimum Drag of Airfoils at Supersonic Speeds. Jour. Aero. Sci., vol. 19, no. 12, Dec. 1952, pp. 813-822.

7. Hayes, Wallace D.: Linearized Supersonic Flow. Rep. AL-222, North American Aviation, Inc., June 18, 1947.

8. Hemke, Paul E.: Drag of Wings With End Plates. NACA Rep. 267, 1927.

9. Lomax, Harvard, and Heaslet, Max. A.: Recent Developments in the Theory of Wing-Body Wave Drag. Paper given at the 24th annual meeting of the IAS, New York, Jan. 23, 1956.

10. Sears, William R.: On Projectiles of Minimum Wave Drag. Quart. Appl. Math., vol. 4, no. 4, Jan. 1947, pp. 361-366.

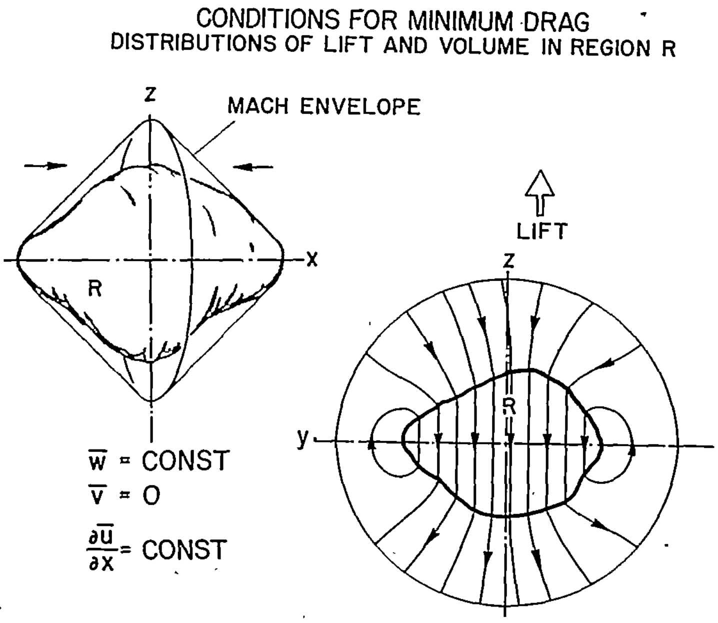


Figure 1

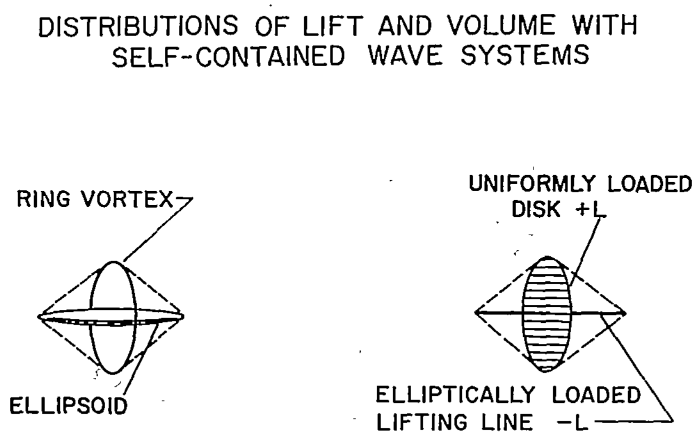
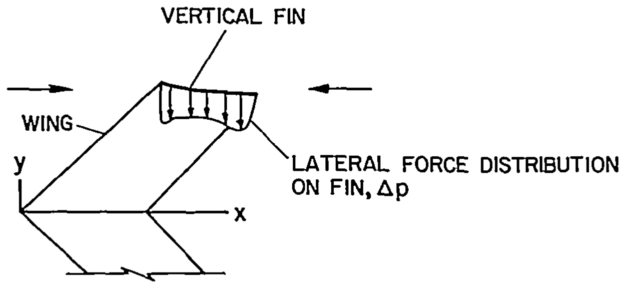


Figure 2

USE OF CONDITION $\bar{v} = 0$ TO DETERMINE OPTIMUM SETTING OF VERTICAL FIN ON WING TIP



$\Delta p_f = \Delta p_r$; LATERAL FORCE DISTRIBUTION UNCHANGED

$v_f = -v_r$; FIN GEOMETRY UNCHANGED

Figure 3

LOWER BOUND FOR WAVE DRAG ASSOCIATED WITH THE REGION R AND THE LIFT L

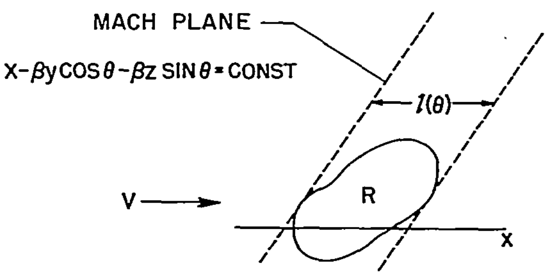


Figure 4

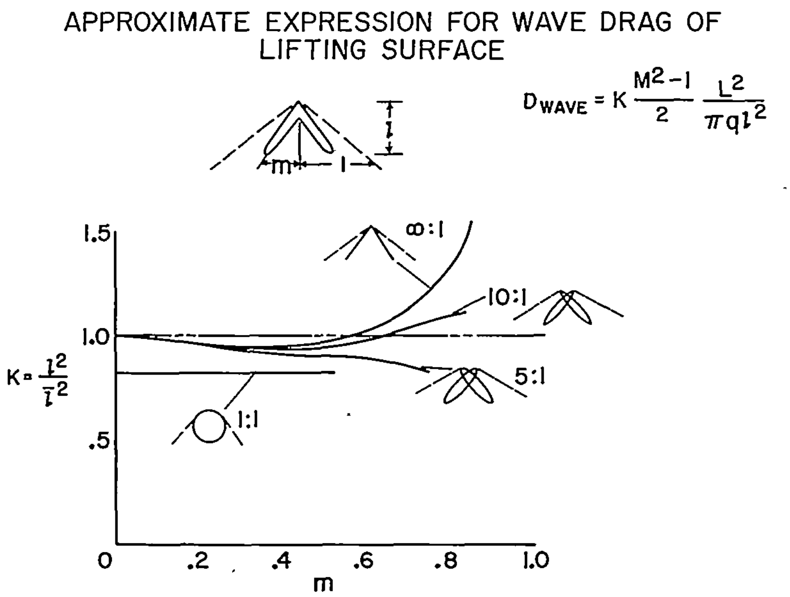


Figure 5

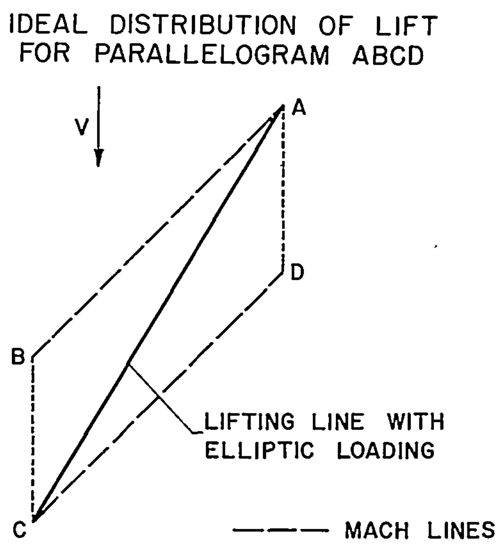


Figure 6